Optimal Binary Predictor
Problem Set

Problem 1: Disease Outbreak Classification

A clinic monitors the number of patients diagnosed with a rare infectious disease each week. Let X denote the number of cases in a week. The clinic wants to detect whether there is an outbreak. Define a binary variable Y such that

$$Y = \begin{cases} 1, & \text{Outbreak} \\ 0, & \text{No outbreak} \end{cases}$$

The conditional distributions of X given Y are discrete Poisson distributions:

$$(X \mid Y = 0) \sim \text{Poisson}(\lambda_0 = 2), \quad (X \mid Y = 1) \sim \text{Poisson}(\lambda_1 = 10)$$

so that

$$P(X = k \mid Y = y) = \frac{\lambda_y^k e^{-\lambda_y}}{k!}, \quad k = 0, 1, 2, \ldots, \quad y \in \{0, 1\}$$

The prior probability of outbreak is $P(Y = 1) = 0.1$. The loss table for the prediction $\hat{Y}$ is:

$\text{loss}(\hat{Y}, Y)$	$Y = 0$	$Y = 1$
$\hat{Y} = 0$	0	10
$\hat{Y} = 1$	1	1

Task:

(i) Suppose the clinic observes $X = 0$ patients in a week. Using Bayes' theorem, explain whether the posterior probability $P(Y = 0 \mid X = 0)$ is close to 0 or 1.

(ii) Determine the likelihood ratio test (LRT) for detecting an outbreak based on X, i.e., identify the smallest integer threshold k such that the clinic should declare an outbreak whenever $X \geq k$.